Physics-Based Modeling of Electromagnetic Parasitic Effects in Interconnects

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Summary

This paper presents a methodology for the analysis of the transient electromagnetic behavior of multiply contacted interconnects.

It is shown that for applications with high switching frequencies and high pulse repetition rates with very steep consecutive current ramps, the usual characterization of interconnects by extracting an inductance matrix based on a stationary current distribution is inadequate and has to be extended or even replaced by the determination of distributed transient fields and the specification of time-dependent characteristic quantities.

Furthermore it is demonstrated that only a full three-dimensional transient analysis under realistic switching conditions can give the necessary insight in the time-dependent behavior of the electric and magnetic fields in- and outside the interconnects, which are the cause of the various distributed parasitic electromagnetic effects.

The method has been implemented in a new finite element simulator based on the C++ library DIFFPACK [1]. This makes it feasible to analyze distributed parasitic effects in realistic interconnect structures with acceptable computational expense within a flexible software environment.

Motivation

The electromagnetic behavior of interconnects is commonly described by an inductance matrix consisting of the self- and mutual inductance coefficients. This is done on the basis of the so-called magnetostatic approximation, assuming a quasi-stationary current distribution in the conductors, which can be easily determined for even complex geometries by solving Poissons equation or, more elaborate, in the time-harmonic domain where also eddy currents and the skin effect are allowed for. For these cases, a well-established method exists, namely the Partial-Element-Equivalent-Circuit method [2], which is based on determining partial inductances by evaluating the well-known Neumann formula.

However, there are applications where the transient character of the electromagnetic dynamical behavior cannot be neglected, such as for very high switching frequencies and high pulse repetition rates with very steep consecutive current ramps. The latter situation is encountered in various application fields and is gaining increasing relevance in electronic systems.

In view of the continuing trend to shortening switching times, we have developed a methodology for the analysis of the full time-dependent electromagnetic behavior of interconnects. It includes the treatment of distributed parasitic effects, in particular time-dependent inductive effects, eddy currents and current crowding phenomena. Originally developed for high power applications, where switching times of some 100 ns or even shorter and switched currents in the range of one kiloampere have become feasible, this analysis method can equally be applied to the field of micro- and nanodevices with much shorter switching times and very much smaller current values but comparable pulse slopes and aspect ratios.

Moreover, it has been demonstrated that only a full three-dimensional transient simulation with realistic pulse forms and switching conditions can reproduce or predictively simulate the real world behavior. The knowledge of the time-dependent electromagnetic fields inside and outside the interconnects and of derived quantities, e. g. the resulting current distributions and their cross-talk, allows the minimization of distributed parasitics and therefore the improvement of the interconnect topography by shape optimization.

Methodological Concepts

We start with the time-dependent Maxwell equations considering time-varying current flows in one or more conductors with unknown distributions of the current density, where either the voltages at the terminals of the conductors (voltage-driven case) or the terminal currents (current-driven case) are given boundary data. The basic configuration is depicted in Fig. 1. Maxwell's equations have to be solved in the so-called quasistationary approximation, where the propagation of electromagnetic waves is neglected.

If the terminal voltages $u_k(t)$ are given, we make use of a magnetic vector potential $\vec{A}$ and an electric scalar potential φ:

$$\vec{B} = curl\vec{A} \quad in\ \Omega_c\ and\ \Omega_n, \tag{1}$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - grad\varphi \quad in\ \Omega_c. \tag{2}$$

In this case the boundary data $u_k(t)$ control the scalar potential at the terminals, while we have homogeneous boundary conditions elsewhere. If the terminal currents $i_k(t)$ are given, the combination of a current vector potential $\vec{T}$ and a magnetic scalar potential Φ is adequate:

$$\vec{J} = curl(\vec{T}_0 + \vec{T}), \tag{3}$$

$$\vec{H} = \vec{T}_0 + \vec{T} - grad\Phi \quad in\ \Omega_c; \tag{4}$$

$$\vec{H} = \vec{T}_0 - grad\Phi \quad in\ \Omega_n. \tag{5}$$

Both approaches are considered in practice. Particular attention has to be paid to the analysis of the adjoint boundary values, terminal currents and voltages, respectively. Based on the above continuous field description

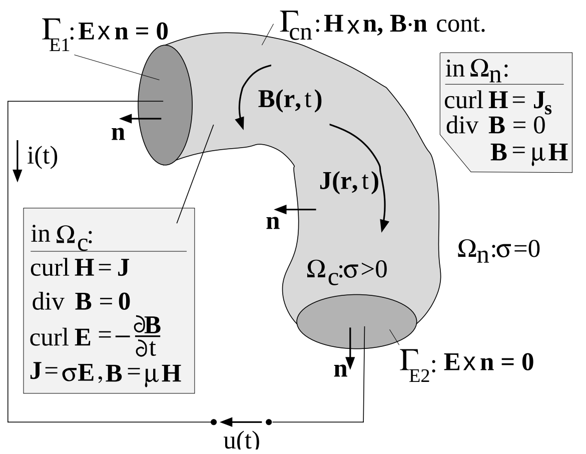

Figure 1: Schematic representation of the quasistationary electromagnetic field problem. The equations to be solved in the conducting and in the non-conducting regions are also shown.

of the problem, we are now ready for focussing on the concept of inductances as a function of time. In the voltage-driven case, we proceed as follows: The governing equation for the vector potential $\vec{A}$ becomes

$$rot\frac{1}{\mu}rot\vec{A} + \sigma\dot{\vec{A}} = -\sigma grad\varphi\,, \tag{6}$$

subjected to Coulomb's gauge $div\,\vec{A} = 0$ as constraint. Normally used to ensure the uniqueness of the magnetic vector potential $\vec{A}$ and for numerical stability, the gauge is additionally used here to decouple the potentials φ and $\vec{A}$. In this way we obtain as governing equation for φ the Laplace equation

$$div\,(\sigma grad\,\varphi) = 0 \tag{7}$$

to calculate the quasi-stationary potential-driven current contribution $\vec{j}_{qs} = -\sigma grad\varphi$ and a kind of diffusion equation

$$\frac{\partial \vec{A}}{\partial t} - \frac{1}{\mu\sigma}\Delta\vec{A} = -\nabla\varphi \tag{8}$$

to determine the vector potential $\vec{A}$. Having calculated the potentials φ and $\vec{A}$, the total current density is obtained by

$$\vec{j} = \vec{j}_{qs} + \vec{j}_{ind} = -\sigma grad\varphi - \sigma\dot{\vec{A}} \tag{9}$$

To deal with problems of multiply contacted interconnects, we build up the quasi-stationary current flow from basis functions, using a separation of space and time variables

$$\vec{j}_{qs}(\vec{r},t) = \sum_{k,\alpha} \vec{j}_{k\alpha}(\vec{r}) I_{k\alpha}(t)\,, \tag{10}$$

where α denotes a single part of an interconnect structure and k a single contact on the interconnect part α. Using the analogue decomposition of the magnetic vector potential,

$$\vec{A}(\vec{r},t) = \sum_{k,\alpha} \int \vec{A}_{k\alpha}(\vec{r}, t-\tau) I_{k\alpha}(\tau) d\tau \tag{11}$$

the resulting current distribution can be expressed as the sum of a source current density $\vec{j}_{qs}$ and an induced current density $\vec{j}_{ind}$, related to each contact electrode $C_{k\alpha}$:

$$\begin{aligned} \vec{j}(\vec{r},t) &= \sum_{k,\alpha} \vec{j}_{k\alpha}(\vec{r}) I_{k\alpha}(t) \\ &\quad -\sigma_\alpha \int \vec{A}_{k\alpha}(\vec{r}, t-\tau) \dot{I}_{k\alpha}(\tau) d\tau \end{aligned} \tag{12}$$

Evaluating the magnetic field energy in terms of the terminal currents $I_{k\alpha}(t)$, we find that a time-dependent inductance matrix can be extracted:

$$\begin{aligned} L_{k\alpha,l\beta}(\tau) &= \int_{\Omega_\alpha} \vec{j}_{k\alpha}(\vec{r}) \vec{A}_{l\beta}(\vec{r},\tau) d^3r \\ &= \frac{1}{\sigma_\alpha} \langle \vec{j}_{k\alpha} | e^{-\mathcal{D}\tau} \vec{j}_{l\beta} \rangle \end{aligned} \tag{13}$$

where $\mathcal{D} = \frac{1}{\mu\sigma}\triangle$.

Based on these quantities and with a view to optimizing the interconnect topography, we are able to define target functionals to assess the quality of a given interconnect structure with respect to, e.g., the switching time delay, signal integrity, signal cross-talk and related quantities of interest.

REFERENCES

[1] Langtangen, H.P.: Computational Partial Differential Equations – Numerical Methods and DIFFPACK Programming. Springer, 2nd edition, 2003.

[2] Ruehli, A.E.: Inductance Calculation in a Complex Integrated Circuit Environment . IBM-Journal of Research and Development (1972), pp. 470–481.